

# GENERATING DYNAMIC MODELS INCLUDING UNCERTAINTY FOR USE IN AIRCRAFT CONCEPTUAL DESIGN

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## **Abstract**

Accurate stability and control derivative information is essential to the configuration designer. As new, non-conventional aircraft are being designed, however, the trusted stability and control estimates usually used in conceptual design may no longer be useful. Using sophisticated analysis to compute every derivative in the highly iterative design environment is not a viable approach either. This paper proposes a method for addressing this dilemma by combining experimental design techniques for model building with vortex lattice aerodynamics for analysis. The careful implementation of this method results in parametric regression equations for three important derivatives as a function of the variables of most interest to the designer (e.g. wing, tail geometry, center of gravity location, etc.). These equations are based on actual analysis and not historical trends. Finally, uncertainty associated with this method is introduced and an initial technique for analyzing the effect of such uncertainty is presented.

## **Introduction**

The usefulness of classic empirical relations for estimating stability and control (S&C) derivatives lies in the fact that the right trends are obtained for most aircraft at little expense. Increasingly, however, this usefulness is being diminished as new classes of vehicles are proposed which are often radically different than previous configurations. For example, the classic S&C relations often require separate calculations of wing, tail, and fuselage contributions to the various force and moments. Future aircraft, however, will likely be highly integrated, and the effect of individual components (if the distinction exists) will be difficult to

discern. If such differences are substantial enough, the use of classic S&C relations may be inappropriate, since most are based on (and validated against) historical data. Yet, configuration designers are increasingly in need of critical S&C information (e.g. static stability, damping characteristics, dynamic response) for many possible alternative designs. In the past, S&C considerations were often neglected to a large degree until after the design space of possible configurations was narrowed to a select few through detailed aerodynamic, structural, and propulsion analyses. When such a down-selection is complete, fairly detailed S&C and Handling Qualities (HQ) assessments may be done to contribute towards the final design. This approach severely limits the role of innovative S&C technologies and can mitigate the positive benefits of such technologies. The obstacle at the heart of this undesirable situation is the significant information requirements (in both quantity and quality) of any non-cursory S&C analysis. Early design settings are very iterative, and the vehicle geometry and associated dynamic characteristics change rapidly. Due to this obstacle, the approach described above of examining a few configurations, testing one or two models in the wind tunnel, and assessing S&C responses on this limited database was adopted. Ref. 1 also recognized this lack of design-oriented S&C analysis.

The purpose of this paper is to present an approach to deal with this dilemma. The approach employs vortex-lattice aerodynamic analysis in conjunction with experimental design techniques to create approximate, parametric models of the key S&C derivatives. This technique maps the vehicle static and dynamic characteristics to the conceptual aircraft "design space". This space is defined as the multidimensional region bounded by minimum and maximum settings of a set of design variables. This type of formulation has the potential to not only provide the needed information for innovative designs, but also to close the gap and improve the communication between the flight control and design groups. Such an improved connection can truly "open the design space" to new solutions previously hidden by the biases of classic designs and

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approaches. Perhaps, on the other hand, it may confirm the validity of such methods.

One of several experimental design techniques through which parametric models can efficiently be formed is the Response Surface Methodology (RSM). A presentation of the detailed theory of experimental design techniques and the exploration of all associated aspects is beyond the scope of this paper. Instead, the reader is referred to Refs. 2, 3 for classic treatments of RSM procedures. In addition, Simpson has provided a good review of alternative model building, or metamodeling, techniques in Ref. 4. Refs. 5, 6 are useful for those interested in recent applications of the RSM to aerospace design problems. A specific objective of this paper is to investigate whether RSM and the regression equations which result from its use is a viable approach to forming parametric S&C models. Validation of typical results against test data is also important. In conjunction with this, a preliminary look is made into the utility of such approximations if they can be successfully constructed.

If such parametric models are found to have utility, the eventual application for them is in a probabilistic aircraft synthesis environment. Aircraft design must be treated probabilistically for a number of reasons. The reason related to the discussions in this paper is the fact that all design models obtained through computer simulation are approximate. Every stability derivative estimate, for example, may have some inherent uncertainties associated with it due to unmodeled effects (e.g. flexibility, non-linear aerodynamics). Even at the conceptual level, neglecting this uncertainty can have a significant consequence. Design decisions based on a rigid aircraft assumption, only to later discover large aeroelastic influences in preliminary design, can dramatically increase the design cost and cycle time. In this environment, the goal is *not optimization*, but *finding the probability of achieving a non-zero feasibility space*. However, the first step must still be forming the most accurate models possible so as to minimize the conservatism built into the uncertainty models. Related work in this area of aircraft model uncertainty was reported in Ref. 7. The emphasis there was on *control system synthesis* and the uncertainties models described were not clearly tied to any rationale related to the analysis. The emphasis here is preparing for *aircraft synthesis* (a complex, multidisciplinary process), where S&C constraints must be examined at the same time as all other disciplines. The goal of the entire probabilistic approach is the achievement of aircraft designs which are robust to modeled uncertainty.<sup>8</sup>

The unique case of a High Speed Civil Transport (HSCT) serves as the example application in this paper.

The fact that an HSCT will likely be a Relaxed Static Stability (RSS) vehicle makes the incorporation of S&C into vehicle synthesis extremely important. RSS can be defined in general terms as the reduction or elimination of inherent static and dynamic vehicle stability requirements.<sup>9</sup> RSS will be present for HSCT due to the well known fact that the aerodynamic center shifts rearward during supersonic flight. Recent industry technology studies have thus concluded that an HSCT airplane will likely be statically unstable longitudinally in low speed due to a desire to design for reduced supersonic trim drag (via placement of the CG such that a neutral or negative static margin occurs).<sup>10</sup> The potential price associated with RSS benefits appears to be low speed stabilization and control authority issues, especially since a fully fueled HSCT will not be able to adjust its CG and high speed maneuverability is not an issue for a transport aircraft. If such savings can be predicted during early design stages, the tail may accordingly be reduced, further reducing drag and weight. Of course, a practical limit for moving the CG rearward is the location of the main landing gear.

### **Approach: Model Building via RSM**

The approach of parametric model generation for static and dynamic S&C characteristics is formulated as an experiment. This is a natural result in light of the use of experimental design techniques. Components of this formulation include: experiment design, experiment execution, model construction, and validation. The expected benefits of the resulting models are:

- A reduced reliance on obsolete historical data
- A capability for rapid design space examination based on actual analysis
- Ease of integration into the aircraft synthesis environment
- Potential usefulness for control law design
- An allowance for direct uncertainty modeling

### **Design the Experiment**

In brief, the RSM comprises a group of statistical techniques for empirical model building and exploitation. The Design of Experiments (DOE) is used within the RSM to efficiently define combinations of design variables which need to be examined to obtain the desired statistical significance.<sup>2</sup> DOE/RSM combines experimental and numerical analysis techniques for the purpose of creating a functional relationship between key design variables and system level responses which are otherwise too expensive to

create. These relationships are manifested as regression equations which are formed from a series of experiments. The earliest applications centered around situations where complex, physical experiments were the only way to determine relationships between experimental factors. In the setting of this paper, the “experiments” are computer simulations. This fact will slightly modify the typical application of these techniques. DOE/RSM provides an alternative to standard parametric approaches in modeling a design space. The technique has a rich and well documented history in many branches of science and engineering.<sup>11,12</sup>

A generic model takes the form of Eq. (1), where  $R$  is the true response,  $f$  is the modeled effects, and  $\varepsilon$  is the combined fit and experimental error.

$$R = f + \varepsilon \quad (1)$$

In this paper, following Refs. 13, 14, a second degree model of the selected responses in  $k$ -variables is assumed to exist. This second degree polynomial is termed a Response Surface Equation (RSE) and takes the general form of:

$$R = b_o + \sum_{i=1}^k b_i x_i + \sum_{i=1}^k b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^k b_{ij} x_i x_j + \varepsilon \quad (2)$$

where,  $b_i$  are regression coefficients for the linear terms,  $b_{ii}$  are coefficients for the pure quadratic terms,  $b_{ij}$  are coefficients for the cross-product terms (second order interactions), and  $b_o$  is the intercept term. A practical limit on  $k$  in the DOE/RSM approach is typically between 10-11. To facilitate the discussion to follow, the components of Eq. (1) are further defined. The  $x_i$  terms are the “main effects”, the  $x_i^2$  terms are the “quadratic effects”, and the  $x_i x_j$  are the “second-order interaction terms”. If a 2nd-order polynomial is found to be inadequate, other forms are possible, such as exponential or logarithmic, through a transformation of both the independent and dependent variables. RSM is often used for *optimization*, thus the attractiveness of the low order polynomial representation. When using RSM to simply *predict* outputs from levels of inputs, whichever model gives the best statistical fit should be used.

### Conduct the Experiment

Several issues are important in conducting the experiments (simulations) called for by the DOE and model selected in the previous step:

a) *Screening*- As a general approach in DOE/RSM, if the number of design variables in the model is above

10, a “screening” is first conducted before the actual model building experiment is run. The objective of the screening is to reduce the number of variables by identifying the most significant contributors to the response. This “screening test” typically uses a two-level fractional factorial DOE to test a linear model, and thus estimates only the main effects of the design variables on the response. However, it allows for the investigation of a high number of variables in order to gain an initial understanding of the problem and the design space. A visual way to see the results of this screening is through a Pareto Chart, examples of which appear in the application results section of this paper.

b) *DOE alternatives*- Following the results of Ref. 8, the three level, face-centered composite DOE is recommended as it has been found to minimize the correlation between model effects which are assumed to be independent (as opposed to the central composite designs). The authors of Ref. 15 support this approach as well.

c) *Unique aspects of RSM+computer simulations*- The error parameter ( $\varepsilon$ ) in Eq.(1) is assumed to have zero experimental error component when computer simulations (as opposed to physical experiments) are being conducted. Computer simulations are *assumed* to be perfectly deterministic in this study (though some may argue this in light of numerical round-off issues). In other words, if a certain series of inputs is given to a computer code 100 times, the same answer is expected each time. The authors in Ref. 4 point out that some typical statistical tests used in RSM for actual experiments are not valid when experimental error is not present. Instead, the figures of merit which should be used are the R-Square value and a test of the model against independent test data sets. The definitions for these measures are given shortly in the validation procedure description.

### Construct the Model- Multivariate Regression

Many commercial software packages are available for the regression of DOE data. The software used in this research is identified in the acknowledgments.

### Validation

The following are some of the key validation procedures which should be applied once the regression is complete:

a) *Check for model completeness*- If there appears to be a pattern in the data which is causing a poor fit, this likely means that an effect is present which was not part of the model. In the case of the 2nd-order model of

Eq.(2), this could mean the presence of a third order effect.

b) *Check the R-square value*- The R-square value is the square of the correlation between the actual and predicted response. Thus, an R-square value of one means that all the fit errors are zero (i.e. a perfect fit).

c) *Check the model against independent data*- A set of random data, separate from the data used to regress the model equation, should be generated and used to test the accuracy of the model (i.e. plot the actual random data against the model prediction values).

## Results

### Design the Experiment: Parametric Derivatives

The four step approach outlined previously is now implemented on a hypothetical HSCT application. The objective of the implementation is to generate efficient, accurate models of key static and dynamic longitudinal responses as a function of aircraft geometry design variables. To illustrate the method, three stability derivatives are chosen as responses:

$$C_{M_\alpha} = \frac{\Delta C_M}{\Delta \alpha} \quad (3)$$

$$C_{M_q} = \frac{\Delta C_M}{(qc/2V)} \quad (4)$$

$$C_{M_{\delta e}} = \frac{\Delta C_M}{\Delta \delta e} \quad (5)$$

where  $C_M$  is the pitching moment coefficient,  $\alpha$  is the angle of attack,  $q$  is the pitch rate,  $c$  is the mean aerodynamic chord,  $V$  is the flight velocity, and  $\delta e$  is the elevator deflection angle.

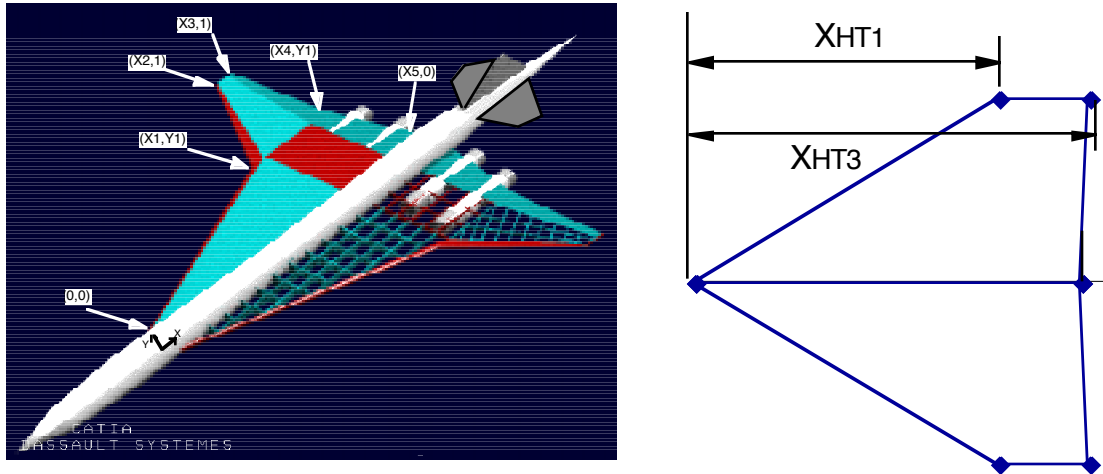
As is well known, Eq. (3) represents a vehicle's longitudinal static stability, Eq. (4) is the pitch damping, and Eq. (5) is a measure of the elevator control power. The selected HSCT geometry variables and their ranges are shown in Table 1 and Figure 1. These ranges establish the design space, outside of which the response models are not valid.

### Vortex Lattice Aerodynamic Analysis

The High Angle of Attack Stability and Control (HASC95) program is used to estimate the longitudinal forces, moments, and associated aerodynamic derivatives.<sup>16</sup> Ref. 1 was valuable in the selection of HASC95 since it compared alternative aerodynamic methods for computing stability derivatives. HASC95 is comprised of a generalized vortex lattice method for force and moment calculations, a semi-empirical strake/wing vortex analysis, and a two dimensional, unsteady, separated flow analogy program for analyzing smooth forebody shapes. The HASC95 code is currently used at both NASA and the Air Force Wright Laboratory and is especially suited for unique configurations (such as an HSCT) or for investigations in flight regimes where non-linear behavior is expected. This program is the "simulation engine" used to generate the regression data for the model building exercise to be described next.

Table 1: The Design Space- HSCT Wing/Tail Variable Ranges

Variable	Minimum	Mid-Point	Maximum	Remarks
X1	1.54	1.615	1.69	Kink LE x-location, normalized by wing semi-span
Y1	0.44	0.51	0.58	Kink LE y-location, normalized by wing semi-span
X2	2.10	2.23	2.36	Tip LE x-location, normalized by wing semi-span
X3	2.40	2.49	2.58	Tip TE x-location, normalized by wing semi-span
X4	2.19	2.275	2.36	Kink TE x-location, normalized by wing semi-span
X5	2.19	2.345	2.50	Root Chord, normalized by wing semi-span
XWING	26%	28%	31%	wing position, % fuselage length
SW	8500	9000	9500	wing ref. area, square feet
XTAIL	82%	84.7%	87.4%	horizontal tail position, % fuselage length
ST	875	922.5	970	horizontal tail ref. area, square feet
XHT1	0.95	1.18	1.20	normalized by HT semi-span
XHT3	1.90	2.00	2.10	normalized by HT semi-span
CG	56%	57.5%	59%	CG, %fuselage



**Figure 1: HSCT Wing and Tail Geometry Definitions**

As with any panel or grid-based aerodynamic method, the complexity of the discretization can influence the results. Unless the paneling scheme is included explicitly in the screening or RSE generation, it must be kept constant for each case in the selected DOE. In this paper, a fairly simple paneling scheme was chosen, an example of which appears in Figure 2. The fuselage and horizontal tail were paneled in a similar way. One must have sufficient complexity to capture the important effects, but asking the code to capture effects for which it was not intended can lead to erroneous instead of improved results.



**Figure 2: Typical Paneling Used for HASC95 Models**

Finally, the following assumptions were made in the analysis:

- *Fuselage*: Except for tapered nose and tail sections, the fuselage width is kept constant at 12 ft. The length is held at 300 ft. for each case.
- *Flight condition*:  $M = 0.3$ , Altitude = 5000 ft. (expected low speed instability makes this regime important for an HSCT)
- *Non-linear effects*: Vortex lift, vortex bursts, and camber effects were not modeled. *HASC95* has the ability to address each, but these capabilities were not pertinent to the goals of this pilot study (though they are certainly pertinent to any detailed study of an HSCT)

### Conduct the Experiment

#### Screening

Based on the variable ranges in Table 1, a two-level fractional DOE in conjunction with HASC95 was used to conduct a screening test for each of the three responses. The screening results are interpreted through a Pareto Chart, which identifies the most significant contributors to the response based on the linear regression equation generated from the DOE data.<sup>17</sup> Bars indicate which variables contribute how much while a line of cumulative contribution tracks the total response. By defining the percentage of contribution desired, the variables to be carried along to the RSE generation can be determined from the array of variables in the Pareto Chart. The charts in Figures 3 through 5 show the relative importance of the variables in order of decreasing importance for the three responses.

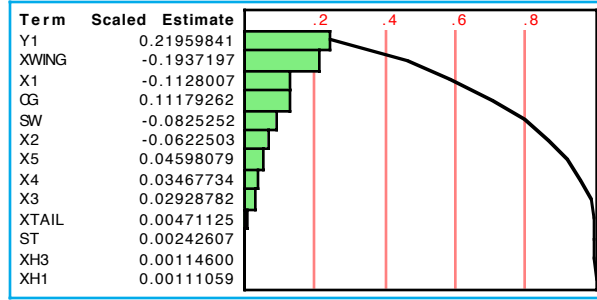


Figure 3: Pareto Chart for  $C_{M\alpha}$

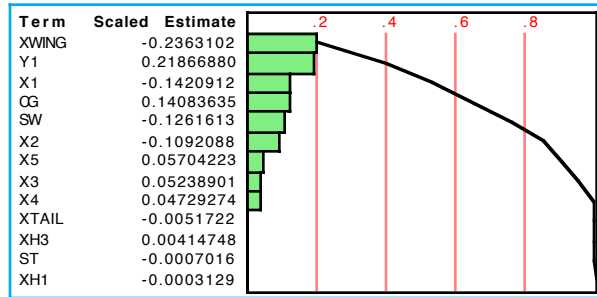


Figure 4: Pareto Chart for  $C_{Mq}$

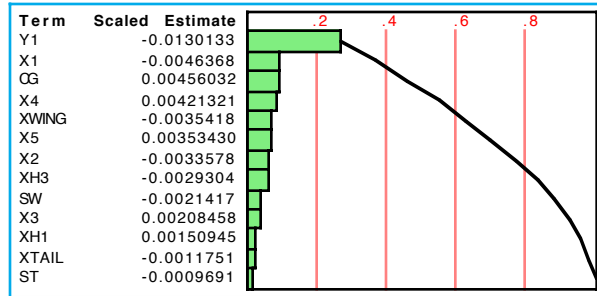


Figure 5: Pareto Chart for  $C_{M\delta e}$

An interpretation of the screening results is given next, and the subset of selected variables for each response is documented in Table 2.

a) The scaled estimates are statistical measures of the contributions of each variable to the response. In general, the screening cut-off occurs at the 80% total contribution point, though engineering judgment should be used when deciding on the subset of important variables. Once the subset is selected, it is passed to the Response Surface Equations (RSEs) generation step.

b) The wing leading edge kink location (X1, Y1), the wing position on the fuselage (XWING), and the

center of gravity (CG) location are the leading contributors in all three responses. This is initially counter-intuitive to classic S&C relations, especially for the pitch damping derivative. Classical relations put forward in both Etkin [Ref. 18] and Nelson [Ref. 19] suggest that the tail contributes around 90% to the  $q$ -derivatives. But, as Etkin points out, the effect of the wing on the  $q$  derivatives may be non-negligible for highly swept or low aspect ratio wings (both of which characterize an HSCT). The large area of an HSCT wing makes wing geometry changes highly influential, primarily due to the shifting of the center of lift. In addition, the tail variable ranges are relatively modest. It was assumed in this study that a horizontal tail is *required* for trim and for pitching moment during wing flap employment. If the tail variable ranges had been extended to include the case of no horizontal tail at all, the influence of these variables surely would be increased. *In light of all this, it is clear that variable ranges must be carefully selected and their effect on a Pareto analysis must not be overlooked.* Finally, though the screening indicates that the variable XTAIL is insignificant, it will be kept in order to allow adjustment of the tail moment arm in later studies where it may have increased significance.

c) Given the geometry, one might wonder why the CG is kept as a variable. The CG must be treated independently (within a prescribed range) because it can vary from flight to flight even for the same aircraft configuration due to fuel management, mission scenario, etc. The eventual use of these equations in aircraft synthesis requires that the CG be input (albeit based on geometry and other factors).

Table 2: Documenting the Screening Results

Variable	Response of Interest		
	$C_{M\alpha}$	$C_{M\delta e}$	$C_{Mq}$
X1	x	x	x
Y1	x	x	x
X2	x	x	x
X3			
X4		x	
X5	x	x	x
XWING	x	x	x
SW	x		x
XTAIL			
ST			
XHT1			
XHT3		x	
CG	x	x	x

### RSE DOE Selection

In a typical application of the RSM, the screening test is used not only to determine the most important parameters, but also to identify the regions of the design space where extremal values of the objective may occur. Subsequently, the design variable ranges of the RSE are focused on that area around the suspected region of good solutions. The quadratic RSE is then used for optimization. However, the purpose of generating RSEs in this paper is *prediction* and not *optimization*. Thus, the same variable ranges are used to define the space, though the space is smaller in dimension after the screening. Using the screening results of Table 2, a three-level face-centered DOE for seven variables is constructed for each of the three responses. The appropriate HASC95 simulations are run and the resulting data is recorded in preparation for regression.

### Construct the Models

RSEs for  $C_{M\alpha}$ ,  $C_{Mq}$ , and  $C_{M\delta e}$  are generated from the DOE data. A useful way to view the equations is through prediction profiles. The profiles, shown in Figure 6, are “snapshots” of the design space corresponding to, in this case, the midpoint settings of all the variables. For clarity of presentation, the design variables ranges are normalized between -1 and 1. The vertical “hairline” for each variable can be manipulated with the resulting prediction for the response updated on the left. Above and below the current predicted value are the minimum and maximum possible outcomes for

that response. The slopes of the profiles depict the sensitivity of the response to the various inputs. *These sensitivities include main effects, second order effects, and interactions.* Most importantly, however, the RSEs can serve as the link to aircraft synthesis, with the understanding that this link is based on carefully planned and conducted analysis as opposed to “rules of thumb”. All derivative values in Figure 6 are expressed as per radian. It is interesting to note that some settings of the variables within the given ranges will give statically unstable designs. The modest values for elevator control power are likely due to the fact that the ratio of tail control surface area to wing area is low for this family of aircraft.

The pitch damping derivative relation is given in Ref. 19 as

$$C_{Mq} = -2(C_{L\alpha_1})\eta V_H \frac{l_t}{c} \quad (6)$$

where  $\eta$  is the tail efficiency parameter and  $V_H$  is the tail volume coefficient, and  $l_t$  is the tail moment arm. For each case executed in constructing the pitch damping RSE, the parameters in Eq.(6) were also tracked, and  $C_{Mq}$  was estimated from Eq.(6). The two predictions were quite different for each case. The mean value for pitch damping for all 79 cases using the RSE was -0.882 while Eq.(6) the mean was -0.005, a significant discrepancy. As mentioned before, the

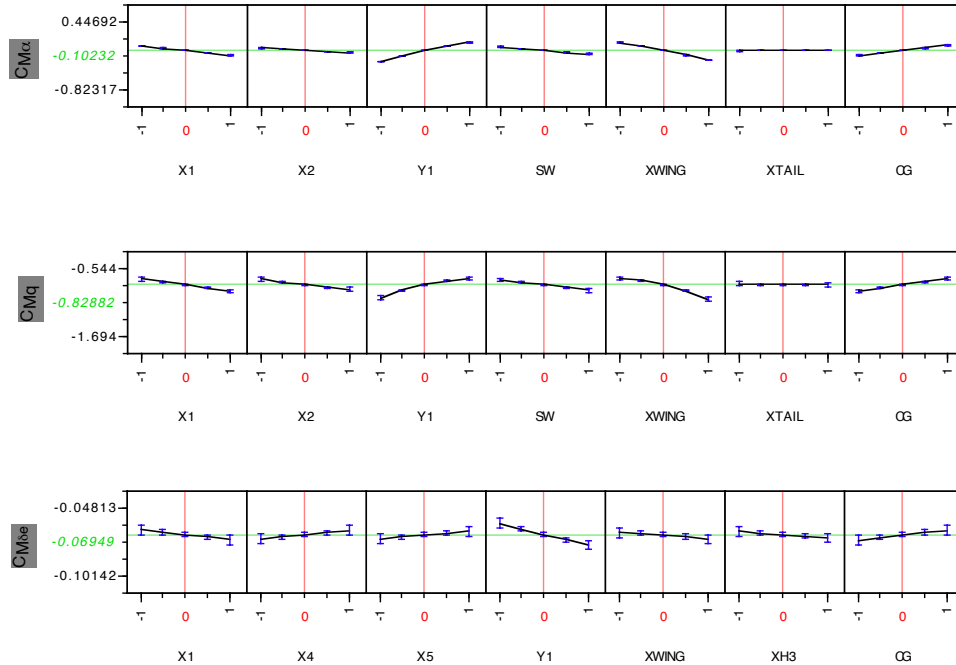


Figure 6: Prediction Profiles for the Three RSEs

discrepancy likely results from the wing-dominated nature of the HSCT force and moment balance. If the experiment is done correctly, the analysis-based RSE would likely be the more appropriate source for predicting the derivative.

#### Validation

The three RSEs performed quite well under validation testing, both in a statistical sense (i.e. the quality of the regression as seen through the R-square) and in an engineering sense (i.e. the quality of the predictions with independent analysis data). The R-square values are shown in Table 3. R-square values of 0.99 and above are considered to be extremely good while anything above 0.95 is still acceptable. Based on this measure, the  $C_{M\alpha}$  and  $C_{Mq}$  RSEs are quite good while the lower R-square for  $C_{M\delta e}$  indicates that there may be some unmodeled effects present.

**Table 3: Statistical Validation: The R-square**

	$C_{M\alpha}$	$C_{Mq}$	$C_{M\delta e}$
R-square	0.9995	0.9963	0.9693

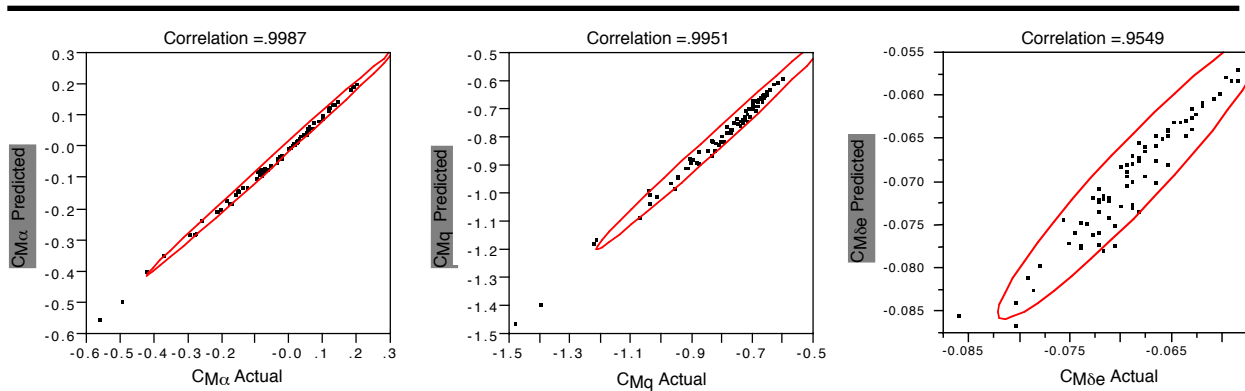
For each derivative, a series of 79 HASC95 cases at random settings of the input design variables were executed. For the same 79 random cases, the RSE predictions were recorded. Figure 7 below shows the RSE prediction plotted against the actual random data points for pitch stability, pitch damping, and elevator effectiveness respectively. These are called correlation plots. If the RSE were a perfect predictor, the correlation would be 1, and the plot would be a line through all the data points from the bottom left to the top right. The correlation values of 0.9987 ( $C_{M\alpha}$ ) and 0.9951 ( $C_{Mq}$ ) are very good. The value of 0.9459 ( $C_{M\delta e}$ )

is less impressive and is related to the lower R-square value for that derivative.

#### Uncertainty

The main purpose of this paper was to describe and illustrate the use of RSM to build prediction equations for key S&C characteristics. However, as mentioned in the introduction, the presence of uncertainty in these estimates has become increasingly important as the aircraft synthesis process moves towards a probabilistic approach.<sup>8</sup> Specifically, the issue of the quantification of fidelity uncertainty needs to be addressed. Analysis fidelity uncertainty in aircraft design can be likened to the uncertainty due to unmodeled dynamics in control law design. For example, estimates of stability derivatives which neglect flexibility effects for an aircraft that will clearly be highly flexible introduces analysis uncertainty. Specifically, elastic effects on supersonic transport stability and control characteristics have been reported and analyzed in the literature.<sup>20,21</sup> The authors in Ref. 1 identified a similar uncertainty in comparing rigid analysis against flight test data (which includes flexibility).

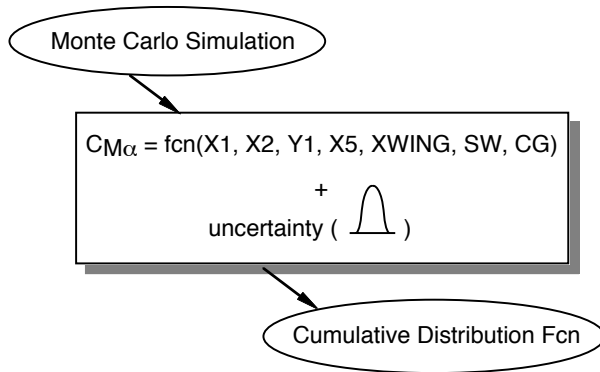
Though the recognition of their presence is clear, means for accounting for them directly within an aircraft design setting is not as clear. Generally, rigid wind tunnel models are used to generate a baseline set of aerodynamic derivatives per configuration and then aeroelastic analysis codes are used to generate estimates of the derivatives including flexibility effects. From this, “rigid-to-flexible” ratios for slope terms and “rigid-to-flexible” increments for absolute values are formed. Apparently, these ratios are used over and over, even if the configuration geometry changes. These ratios and increments have been found to be quite substantial, and thus critical to inclusion in the analysis and decision making process in early design phases.



**Figure 7: Correlation Plots- Predicted Values Against Independent Data**

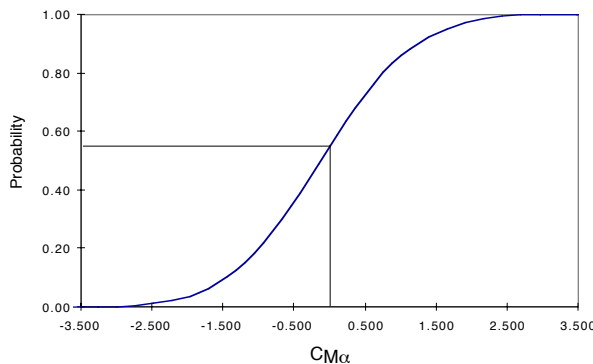


As an initial alternative to these ratios, a method is proposed here which makes use of the parametric models developed earlier in this paper. Specifically, the RSE for  $C_{M\alpha}$  is used to compute the probability of obtaining statically stable designs in the presence of analysis fidelity over the design space. This is accomplished through a Monte Carlo simulation. The RSE provides the mean value and a standard normal density function models the analysis fidelity uncertainty (for example due to flexibility effects). This method is depicted in Figure 8.



**Figure 8 : Probabilistic Design Space Search**

Much more elaborate and detailed scenarios and distributions can be selected if good information on the structure of the uncertainty is known. For illustration purposes in this paper, the standard normal distribution is sufficient. The inputs to the RSE (i.e. the design variables listed in Table 2) are also given distributions to allow the entire design space to be sampled. The Monte Carlo simulation results in the cumulative distribution function (CDF) in Figure 9.



**Figure 9 : CDF- Probability of Stable Designs**

Figure 9 shows that the probability of the occurrence of a stable design in the modeled space is about 0.55. Note that this probability is due to the analysis fidelity uncertainty and the effect of the design variable combinations themselves. This method appears to quickly and efficiently give the designer insight into the nature of the design space and the likely activity of important S&C constraints.

## Conclusions

Preliminary results indicate that experimental design techniques such as the Response Surface Method are useful for generating parametric models for key stability and control characteristics for use in aircraft conceptual design. Such models are certainly needed as classic, historically based stability and control derivative estimates become increasingly obsolete. The fundamentals, procedures, and cautions associated with using RSM for this purpose were presented. For a High Speed Civil Transport example application, the combined use of RSM and vortex lattice aerodynamics resulted in response surface equations for three important stability derivatives: the static pitch stability, the pitch damping, and the elevator control power. These equations are a function of key vehicle configuration variables such as the wing and tail geometry. The equations were validated against independent data and found to be extremely accurate. Finally, an uncertainty model to account for inaccuracies in the analysis was constructed and resulted in a cumulative distribution function for the static pitch stability parameter. This distribution indicated the probability of having a statically stable configuration, given the uncertainty definition and the modeled design space.

## Acknowledgments

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